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Building bridges between neural models and complex decision making behaviour

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Abstract

Diffusion processes, and their discrete time counterparts, random walk models, have demonstrated an ability to account for a wide range of findings from behavioural decision making for which the purely algebraic and deterministic models often used in economics and psychology cannot account. Recent studies that record neural activations in non-human primates during perceptual decision making tasks have revealed that neural firing rates closely mimic the accumulation of preference theorized by behaviourally-derived diffusion models of decision making.

This article bridges the expanse between the neurophysiological and behavioural decision making literatures specifically, decision field theory [Busemeyer, J. R. & Townsend, J. T. (1993). Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review*, *100*, 432–459], a dynamic and stochastic random walk theory of decision making, is presented as a model positioned between lower-level neural activation patterns and more complex notions of decision making found in psychology and economics. Potential neural correlates of this model are proposed, and relevant competing models are also addressed.
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The decision processes of sensory-motor decisions are beginning to be fairly well understood both at the behavioural and neural levels. For the past ten years, neuroscientists have been using multiple cell recording techniques to examine spike activation patterns in rhesus monkeys during simple decision making tasks (Britten, Shadlen, Newsome, & Movshon, 1993). In a typical experiment, the monkeys are presented with a visual motion detection task which requires them to make a saccadic eye movement to a location indicated by a noisy visual display, and they are rewarded with juice for correct responses. Neural activity is recorded from either the middle temporal area (an extrastriate visual area), lateral intraparietal cortex (which plays a role in spatial attention), the frontal eye fields (FEF), or superior colliculus (SC, regions involved in the planning and implementation of eye movements, respectively).

The typical findings indicate that neural activation regarding stimulus movement information is accumulated across time up to a threshold, and a behavioural response is made as soon as the activation in the recorded area exceeds the threshold (see Gold and Shadlen (2000), Mazurek, Roitman, Ditterich, and Shadlen (2003), Ratcliff, Cherian, and Segraves (2003), Schall (2003), Shadlen and Newsome (2001) for examples). Because areas such as FEF and SC are thought to implement the behaviour of interest (in this example, saccadic eye movements), a conclusion that one can draw from these results is that the neural areas responsible for planning or carrying out certain actions are also responsible for deciding the action to carry out, a decidedly embodied notion.

Mathematically, the spike activation pattern, as well as the choice and response time distributions, can be well described by what are known as diffusion models (see Smith and Ratcliff (2004) for a summary). Diffusion models can be viewed as stochastic recurrent neural network models, except that the dynamics are approximated by linear systems. The linear approximation is important for maintaining a mathematically tractable analysis of systems perturbed by noisy inputs. In addition to these neuroscience applications, diffusion models (or their discrete time, random walk, analogues) have been

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used by cognitive scientists to model performance in a variety of tasks ranging from sensory detection (Smith, 1995), and perceptual discrimination (Laming, 1968; Link & Heath, 1975; Usher & McClelland, 2001), to memory recognition (Ratcliff, 1978), and categorization (Ashby, 2000; Nosofsky & Palmeri, 1997). Thus, diffusion models provide the potential to form a theoretical bridge between neural models of sensory-motor tasks and behavioural models of complex-cognitive tasks.

The purpose of this article is to review applications of diffusion models to human decision making under risk with conflicting objectives. Traditionally, the field of decision making has been guided by algebraic utility theories such as the classic expected utility model (von Neumann & Morgenstern, 1944) or more complex variants such as cumulative prospect theory (Tversky & Kahneman, 1992). However, a number of paradoxical findings have emerged in the field of human decision making that are difficult to explain by traditional utility theories. We show that diffusion models provide a cogent explanation for these complex and puzzling behaviours. First, we describe how diffusion models can be applied to risky decisions with conflicting objectives; second, we explain some important findings using this theory; and finally, we compare this theory with some alternate competing neural network models.

1. Risky decisions with multiple objectives

Consider the following type of risky decision with multiple objectives. Suppose a commander is suddenly confronted by an emergency situation, and must quickly choose one action from a set of J actions, labelled here as $\{A_1, \ldots, A_j, \ldots, A_J\}$. The payoff for each action depends on one of set of K uncertain states of the world $\{X_1, \ldots, X_k, \ldots, X_K\}$. The payoff produced by taking action j under state of the world k is denoted x_{jk}. Finally, each payoff can be described in terms of multiple competing objectives (e.g. one objective is to achieve the commander's mission while another objective is to preserve the commander's resources).

According to the 'rational' model (Savage, 1954; von Neumann & Morgenstern, 1944), the decision maker should choose the course of action that maximizes expected utility: $EU(A_j) = \sum_k p_k \cdot u(x_{jk})$, where p_k is the probability of state X_k and $u(x_{jk})$ is the utility of payoff x_{jk}. Psychological variants of expected utility theory modify the classic model by replacing the objective probabilities with subjective decision weights (e.g. Birnbaum, Coffey, Mellers, and Weiss (1992), Tversky and Kahneman (1992)).

2. Decision field theory

An alternate approach towards explaining risky choice behaviour involves the application of diffusion processes, via decision field theory. We have applied diffusion models to a broad range of results including findings from decision making under uncertainty (Busemeyer & Townsend, 1993), multi-attribute decisions (Diederich, 1997), multi-alternative choices (Roe, Busemeyer, & Townsend, 2001) and multiple measures of preference (Johnson & Busemeyer, 2005). The basic assumptions of the model are summarized below.

2.1. Basic assumptions

Define $P(t)$ as a J dimensional preference state vector, and each coordinate, $P_j(t)$, represents the preference state for one of the J actions under consideration. The preference states may range from positive (approach states) to negative (avoidance states), and the magnitude of a preference state represents the strength of the approach–avoidance tendency. The initial state at the beginning of the decision process, $P(0)$, represents preferences before any information about the actions is considered, such as memory from previous experience with a decision problem ($\sum_j P_j(0) = 0$). For novel decisions, the initial states are all set equal to zero (neutral), $P_j(0) = 0$ for all j. The change in state across a small time increment h is denoted by $dP(t) = P(t) - P(t-h)$.

During deliberation the preference state vector evolves according to the following linear stochastic difference equation ($t = n \cdot h$ and $n = 1, 2, \ldots$)

$$dP(t) = -h \cdot \mathbf{\Gamma} \cdot P(t-h) + V(t) \tag{1a}$$

or equivalently

$$\begin{aligned} P(t) &= (\mathbf{I} - h \cdot \mathbf{\Gamma}) \cdot P(t-h) + V(t) \\ &= \boldsymbol{S} \cdot P(t-h) + V(t) \end{aligned} \tag{1b}$$

where $\boldsymbol{S} = (\mathbf{I} - h \cdot \mathbf{\Gamma})$ is a $J \times J$ feedback matrix, $V(t)$ is a J dimensional stochastic input, and $\mathbf{I}$ is the identity matrix. The solution to this linear stochastic difference equation equals

$$P(t) = \sum_{\tau=0}^{n-1} \boldsymbol{S}^{\tau} V(t - \tau h) + \boldsymbol{S}^{n} P(0). \tag{2}$$

As $h \to 0$, this system approximates an Ornstein–Uhlenbeck diffusion process (see Busemeyer and Diederich (2002), Busemeyer and Townsend (1992)). If the feedback matrix is set to $\boldsymbol{S} = \mathbf{I}$ (i.e. $\mathbf{\Gamma} = 0$), then the Ornstein–Uhlenbeck model reduces to a Wiener diffusion process.

The feedback matrix $\mathbf{\Gamma}$ contains self feedback coefficients, $\gamma_{ii} = \gamma$ are all equal across the diagonals, as well as symmetrical lateral inhibitory connections $\gamma_{ij} = \gamma_{ji}$. The magnitudes of the lateral inhibitory connections are assumed to be inversely related to the conceptual distance between the actions (examples are discussed later). The preference state vector, $P(t)$, remains bounded as long as the eigenvalues of $\boldsymbol{S}$ are less than one in magnitude. Lateral inhibition is commonly used in competitive neural network systems (Grossberg, 1988; Rumelhart & McClelland, 1986).

The stochastic input vector, $V(t)$, is called the valence, and each coordinate, $V_j(t)$, represents avoidance (when activations are negative) or approach (when activations are positive) forces on the preference state for the jth action. The valence vector is decomposed into three parts as follows:

$$V(t) = \boldsymbol{C} \cdot \boldsymbol{M} \cdot W(t), \tag{3}$$

where $\boldsymbol{C}$ is a $J \times J$ contrast matrix, $\boldsymbol{M}$ is a $J \times K$ value matrix, and $W(t)$ is a $K \times 1$ stochastic attention weight vector. Each element, m_{jk}, of the value matrix $\boldsymbol{M}$ represents the affective evaluation of each possible consequence k for each action j. The product of the stochastic attention weight vector with the value matrix, $\boldsymbol{M} \cdot W(t)$, produces a weighted average evaluation for each action at any particular moment. The contrast matrix $\boldsymbol{C}$ has elements $c_{ij} = 1$ for $i = j$ and $c_{ij} = -1/(J-1)$ for $i \neq j$, which are designed to compute the advantage or disadvantage of each action relative to the average of the other actions at each moment. (Note that $\sum_j c_{ij} = 0$ implies that $\sum V_j(t) = 0$.)

The weight vector, $W(t)$, is assumed to fluctuate from moment to moment, representing changes in attention to the uncertain states across time. More formally, the attention weights are assumed to vary according to a stationary stochastic process with mean $E[W(t)] = \boldsymbol{w} \cdot h$, and variance–covariance matrix $\mathrm{Cov}[W(t)] = \boldsymbol{\Psi} \cdot h$.[1] The mean weight vector, $\boldsymbol{w}$, is a $K \times 1$ vector, and each coordinate, $w_k = E[W_k(t)]$, represents the average proportion of time spent attending to a particular state, X_k, with $k \in \{1, \ldots, K\}$.

2.2. Derivations

These assumptions lead to the following implications. The mean input valence equals $E[V(t)] = \boldsymbol{\delta} \cdot h = (\boldsymbol{C} \cdot \boldsymbol{M} \cdot \boldsymbol{w}) \cdot h$, and the variance–covariance matrix of the input valence equals $\mathrm{Cov}[V(t)] = \boldsymbol{\Phi} \cdot h = (\boldsymbol{C M \Psi M' C'}) \cdot h$. The mean preference state vector equals

$$\boldsymbol{\xi}(t) = E[P(t)] = \sum_{\tau=0}^{n-1} \boldsymbol{S}^{\tau} \delta \cdot h + \boldsymbol{S}^{n} P(0)$$
$$= (\mathbf{I} - \boldsymbol{S})^{-1}(\mathbf{I} - \boldsymbol{S}^{n}) \cdot \boldsymbol{\delta} \cdot h + \boldsymbol{S}^{n} P(0). \quad (4)$$

As $t \to \infty$, $\boldsymbol{\xi}(\infty) = (\mathbf{I} - \boldsymbol{S})^{-1} \cdot \boldsymbol{\delta} \cdot h$ so that the mean preference state is a linear transformation of the mean valence. The variance–covariance of the preference state vector equals

$$\boldsymbol{\Omega}(t) = \mathrm{Cov}[P(t)] = h \cdot \sum_{\tau=0}^{n-1} \boldsymbol{S}^{\tau} \boldsymbol{\Phi} (\boldsymbol{S}^{\tau})'. \quad (5)$$

For the special case where $\boldsymbol{\Phi} = \phi^2 \cdot \mathbf{I}$, then $\boldsymbol{\Omega}(t) = h \cdot \phi^2 \cdot (\mathbf{I} - \boldsymbol{S}^2)^{-1}(\mathbf{I} - \boldsymbol{S}^{2n})$; and as $t \to \infty$, then $\boldsymbol{\Omega}(\infty) = h \cdot \phi^2 \cdot (\mathbf{I} - \boldsymbol{S}^2)^{-1}$.

Finally, if it is assumed that the attention weights change across time according to an independent and identically distributed process, then it follows from the central limit theorem that the distribution for the preference state vector will converge in time to a multivariate normal distribution with mean $\boldsymbol{\xi}(t)$ and covariance matrix $\boldsymbol{\Omega}(t)$. The derivation for the choice probabilities depends on the assumed stopping rule for controlling the decision time, which is described next.

[1] Preference is a stochastic process which should converge to a diffusion process as $h \to 0$. The properties of preference depend upon the valence which is a random variable because the attention weights are random. In order for preference to converge to a diffusion process, the mean and variance must be proportional to h. And because valence is a linear transformation of the weights, the mean and variance of the weight vector must be proportional to h.

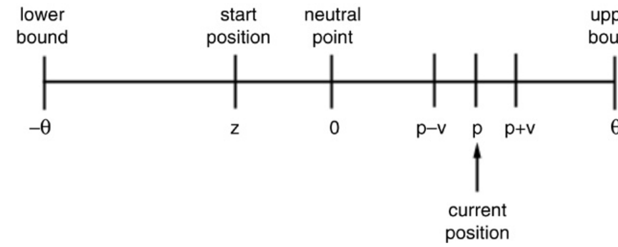

Fig. 1. Representation of the binary decision process in decision field theory. The decision process begins at the start position z (often $z = 0$, the neutral point) and preference is accumulated for either of the two options. The current level of accumulated preference p is incremented by positive values of valence v and decremented by negative values. One option is chosen when preference exceeds the upper bound θ, and the alternate option is chosen when preference exceeds the lower bound.

Externally controlled stopping task. In this first case, the stopping time is fixed at time T, and the action with the maximum preference state at that time is chosen. For a binary choice, allowing $h \to 0$ produces the following equation (Busemeyer & Townsend, 1992):

$$\Pr[A_1 \mid \{A_1, A_2\}] = F\left(\frac{\mathrm{e}^{-\alpha \cdot T} \cdot z + (1 - \mathrm{e}^{-\alpha \cdot T}) \cdot (\delta/\alpha)}{\frac{\sigma}{\sqrt{2 \cdot \alpha}} \cdot \sqrt{1 - \mathrm{e}^{-2 \cdot \alpha \cdot T}}} \right) \quad (6)$$

where F is the standard normal cdf, T is the fixed time, $z = P_1(0)$, $\delta = \lim_{h \to 0} E[V_1(t)]/h = \sum (m_{1k} - m_{2k}) \cdot w_k$, $\sigma^2 = \lim_{h \to 0} \mathrm{Var}[V_1(t)]/h = \phi_1^2 + \phi_1^2 - 2 \cdot \phi_{12}$, and $\alpha = (\gamma_{11} + \gamma_{12})$. If there is no initial bias, $z = 0$, then the binary choice probability is an increasing function of the ratio (δ/σ). Also as $T \to \infty$, the binary choice probability is determined solely by the ratio $\sqrt{\frac{2}{\alpha}} \cdot \left(\frac{\delta}{\sigma}\right)$.

Internally controlled stopping task. In this case, rather than a fixed deliberation time, there is some sufficient level of preference required to make a choice. The deliberation process continues until one of the preference states exceeds this threshold, θ, and the first to exceed the threshold is chosen (see Fig. 1). For a binary choice, allowing $h \to 0$ produces the following equation (Busemeyer & Townsend, 1992):

$$\Pr[A_1 \mid \{A_1, A_2\}] = \frac{\int_{-\theta}^{z} \exp\left(\frac{\alpha \cdot y^2 - 2 \cdot \delta \cdot y}{\sigma^2}\right) \mathrm{d}y}{\int_{-\theta}^{\theta} \exp\left(\frac{\alpha \cdot y^2 - 2 \cdot \delta \cdot y}{\sigma^2}\right) \mathrm{d}y}. \quad (7)$$

Here $z = P_1(0)$ is the initial preference state, θ is the threshold bound, $\delta = \sum (m_{1k} - m_{2k}) \cdot w_k$, $\sigma^2 = \phi_1^2 + \phi_1^2 - 2 \cdot \phi_{12}$, and $\alpha = (\gamma_{11} + \gamma_{12})$. For $\alpha < (\delta/\theta)$, the binary choice probability is an increasing function of the ratio (δ/σ) (see Busemeyer and Townsend (1992), proposition 2).

3. Connections with neuroscience

According to decision field theory, lateral inhibition is critical for producing a variety of robust empirical phenomena (see Section 5.2). The locus of this lateral inhibition may lie within the basal ganglia, which have been implicated in decision behaviour through their feedback loops to key cortical areas (Middleton & Strick, 2001). Moreover, Schultz et al.

(1995) observed that dopaminergic neurons afferent to the basal ganglia fire in concert with reliable predictors of reward (see also Gold (2003), Hollerman and Schultz (1998), and Schultz (1998, 2002)). Together these findings support the notion that the basal ganglia have an important function in decision behaviour.

Knowledge of the basal ganglia architecture should enhance our understanding of the role of lateral inhibition within cortico-striatal loops. In particular, we are concerned with two substructures in the basal ganglia, the globus pallidus internal segment (GPi) and the striatum. Within the cortico-striatal loops, axons from the cortex enter into the basal ganglia via the striatum, which then projects to GPi, which in turn projects to the thalamus which sends afferent connections to the cortical area from which it arose, creating a looped circuit of neural communication (Middleton & Strick, 2001).

The striatum consists of approximately 95% GABAergic medium spiny neurons (Gerfen & Wilson, 1996). Because GABA is an inhibitory neurotransmitter, these are inhibitory neurons. Additionally, these striatal neurons have extensive local arborization of dendrites and axons, creating a network of distance dependent laterally inhibited neurons (Wickens, 1997; Wilson & Groves, 1980). Striatal neurons have inhibitory connections to GPi, the output mechanism of the basal ganglia complex. GPi consists of tonically active neurons (TANs), which exert their effects by continuously firing, so as to relentlessly inhibit post-synaptic neurons in the thalamus; only by inhibition of inhibitory neurons can neurons cast off the shackles of TANs. Inhibition of GPi by striatal neurons releases the thalamus to signal the frontal cortex to engage in the action preferred by striatal neurons. This process is known as thalamic disinhibition.

Naturally, when one option appears in isolation, the lack of lateral inhibition from competing alternatives will enable it to quickly inhibit corresponding GPi neurons[2]; however, multiple competing alternatives arouse lateral inhibition. Much as the edge detectors in the bipolar cells within the retina (which also employ distance dependent lateral inhibition) are tuned to contrasts and thereby enhance differences, these basal ganglia cells can also be thought of as focusing on contrasts between alternatives. Thus, the local contrasts between a dominated alternative are effectively magnified by striatal units, giving more proximal alternatives an advantage over more distal options where contrasts are less magnified. The negated inhibition employed by decision field theory appears to mimic this magnification of local differences.

On the other hand, it might be reasonable to suggest that the negated inhibition corresponds with thalamic disinhibition, or striatal inhibition of GPi (see Busemeyer, Townsend, Diederich, and Barkan (2005), Vickers and Lee (1998) for related proposals). While distributed representations of alternatives via striatal neurons engage in lateral inhibition, they also send inhibitory connections to tonically active GPi neurons. As information favouring one alternative begins to weaken, representative neurons send less lateral and pallidal (i.e. to the GPi) inhibition. Competitive striatal neurons representing alternate options are thus less inhibited, enabling them to increase inhibition of both the weakened alternative and GPi neurons representing their specific alternative. This might look very much like the negated inhibition employed by decision field theory.

4. The bridge

Decision field theory is based on essentially the same principles as the neural models of sensory-motor decisions (e.g. Gold and Shadlen (2001, 2002)—preferences accumulate over time according to a diffusion process. So how does this kind of model relate to expected utility models? To answer this question, it is informative to take a closer look at a simple version of the binary choice model in which $\boldsymbol{S} = \mathbf{I}$ (i.e. $\boldsymbol{\Gamma} = 0$). In this case:

$$P_1(t) = P_1(t-h) + V_1(t) = z + \sum_{\tau=1}^{n-1} V_1(\tau \cdot h) \tag{8}$$

where $z = P_1(0)$. Recall that $E[V(t)] = h \cdot \boldsymbol{C} \cdot \boldsymbol{M} \cdot w = h \cdot \boldsymbol{\delta}$, and for $J = 2$ alternatives, $\boldsymbol{C} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $\boldsymbol{\delta} = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} \delta \\ -\delta \end{bmatrix}$. The expectation for the first coordinate equals:

$$\begin{aligned} E[V_1(t)] = \delta \cdot h &= \left(\sum w_k \cdot m_{1k} - \sum w_k \cdot m_{2k}\right) \cdot h \\ &= (\mu_1 - \mu_2) \cdot h \end{aligned} \tag{9}$$

where $w_k = E[W_k(t)]$ is the mean attention weight corresponding to the state probability, p_k, in the classic expected utility model, $\mu_j = \sum w_k \cdot m_{jk}$ corresponding to the expected utility of the jth option, and right hand side of Eq. (9) corresponds to a difference in expected utility. Thus the stochastic element, $V_1(t)$, can be broken down into two parts: its expectation plus its stochastic residual, with the latter defined as $\varepsilon(t) = V_1(t) - \delta \cdot h$. Inserting these definitions into Eq. (8) produces

$$P_1(t) = (\mu_1 - \mu_2) \cdot t + \left(z + \sum_{\tau=1}^{n-1} \varepsilon(\tau \cdot h)\right). \tag{10}$$

According to Eq. (10), the preference state is linearly related to the mean difference in expected utilities plus a noise term.

By comparing Eqs. (8) and (10), one can see that it is clearly unnecessary to assume that the neural system actually computes a sum of the products of probabilities and utilities so as to compute expected utility when choosing between alternatives; instead, an expected utility estimate simply emerges from temporal integration. According to this analysis, neuroeconomists should not be looking for locations in the brain that combine probabilities with utilities. The key question to ask is what neural circuit in the brain carries out this temporal integration.

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[2] It is important to note that rarely are we faced with one option in isolation, as we can seemingly always choose the 'not' option, i.e. we can choose to buy or to not buy; the notion of 'one option' is rather artificial. An alternate construal might arise when the option to choose dominates the 'not' option; this may be what is generally implied when it is said that there is only one option.

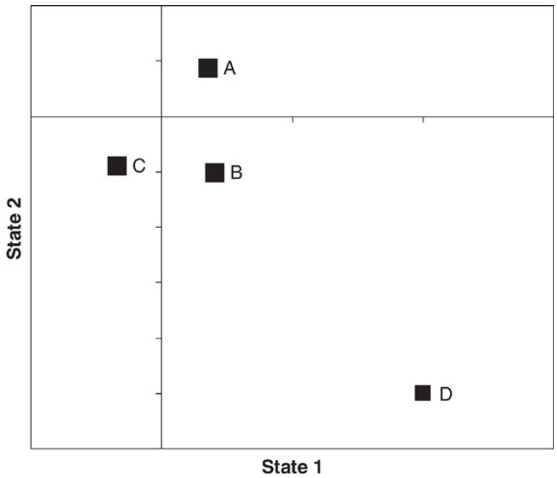

Fig. 2. Comparability effects. Four actions (A, B, C, & D) can be taken, and the payoffs that are obtained depend on whether State 2 or State 2 occurs. If State 2 occurs, choice A would yield the highest payoff and D would yield the lowest. If State 1 occurs, D would give the highest payoff whereas C would provide the lowest. Binary choice comparisons between the available actions elicit violations of strong stochastic transitivity and order dependence. Unlike most utility models, decision field theory is able to explain these violations.

5. Gains in explanatory power

Decision field theory, being a dynamic and stochastic model, seems more complex than the traditional deterministic and algebraic decision theories. However, it adds explanatory power that goes beyond the capabilities of traditional decision theories. Generally, it allows for predictions regarding deliberation time, strength of preference, response distributions, and process measures that are not possible with static, deterministic approaches. More specifically, there are several important findings which, although puzzling for traditional decision theories, are directly explained by decision field theory; some of these are reviewed next.

5.1. Similarity effects on binary choice

Busemeyer and Townsend (1993) survey robust empirical trends that have challenged utility theories but that are easily accommodated by applying decision field theory to binary choices. We consider here one example, illustrated using the four choice options (actions) shown in Fig. 2. In this example, assume that there are two equally likely states of the world, labelled State 1 and State 2. The horizontal axis represents the payoffs that are realized if State 1 occurs, and the vertical axis represents the payoffs that are realized if State 2 occurs. For example, if State 1 occurs, then action D pays a high value and action B pays a low value; but if State 2 occurs, then B pays a high value and D pays a low value. Action A has a very small disadvantage relative to action B if State 1 occurs, but it has a more noticeable advantage over B if State 2 occurs. Action C has a noticeable disadvantage relative to B if State 1 occurs, but it has a very small advantage relative to B if State 2 occurs.

In this type of situation, a series of experiments have produced the following general pattern of results (see Busemeyer and Townsend (1993), Erev and Barron (2005) Mellers and Biagini (1994)). On the one hand, action B seems clearly better than action C, and so it is almost always chosen over action C; on the other hand, action B seems clearly inferior to action A, and so it almost never chosen over action A. Things are not so clear when action D is considered, for this case involves large advantages and disadvantages for D depending on the state of nature. Consequently, action D is only chosen a little more than half the time over action C, and action D is chosen a little less than half the time over action A. Finally, when given a binary choice between B versus D, people are equally likely to choose each action. In sum, the following pattern of results is generally reported:

$$\begin{aligned}\Pr[B \mid \{B, C\}] &> \Pr[D \mid \{D, C\}] \geq \Pr[D \mid \{B, D\}] \\ &= .50 \geq \Pr[D \mid \{D, A\}] > \Pr[B \mid \{B, A\}].\end{aligned}$$

This pattern of results violates the choice axioms of strong stochastic transitivity and order independence (Tversky, 1969).

The pattern is very difficult to explain using a traditional utility model. The probability of choosing action X over Y is a function of the expected utilities assigned to each action:

$$\Pr[X \mid \{X, Y\}] = F[EU(X), EU(Y)],$$

with F a strictly increasing function of the first argument, and a strictly decreasing function of the second.[3] Now the first inequality implies,

$$\begin{aligned}\Pr[B \mid \{B, C\}] &= F[EU(B), EU(C)] \\ &> F[EU(D), EU(C)] = \Pr[D \mid \{D, C\}],\end{aligned}$$

which also implies that $EU(B) > EU(D)$. But the latter in turn implies

$$\begin{aligned}\Pr[B \mid \{B, A\}] &= F[EU(B), EU(A)] \\ &> F[EU(D), EU(A)] = \Pr[D \mid \{D, A\}],\end{aligned}$$

which is contrary to the observed results. In other words, the utilities would have to have the reverse order, $EU(D) > EU(B)$, to account for the second inequality.

Decision field theory provides a simple explanation for this violation of order independence. For two equally likely states of nature, attention switches equally often from one state to another so that $w_1 = .50 = w_2$. When given a binary choice between two actions X and Y, then $E[V_x(t)] = \delta = (\mu_X - \mu_Y) = w_1 \cdot (m_{X1} - m_{Y1}) + w_2 \cdot (m_{X2} - m_{Y2})$ and $\mathrm{Var}[V_x(t)] = \sigma^2 = w_1 \cdot (m_{X1} - m_{Y1})^2 + w_2 \cdot (m_{X2} - m_{Y2})^2 - (\mu_X - \mu_Y)^2$. Based on the values shown in Fig. 2, the mean differences for the four pairs are ordered as follows:

$$\begin{aligned}+\delta &= \mu_B - \mu_C = \mu_D - \mu_C > \mu_B - \mu_D \\ &= 0 > \mu_B - \mu_A = \mu_D - \mu_A = -\delta.\end{aligned}$$

This alone does not help explain the pattern of results. The variances for the four pairs are ordered as follows from high to low:

$$\sigma_H^2 = \sigma_{DC}^2 = \sigma_{DA}^2 > \sigma_{BC}^2 = \sigma_{BA}^2 = \sigma_L^2.$$

[3] This argument holds even if the utility of an action is computed using decision weights for the outcomes rather than the objective state probabilities.

By itself, this also does not explain the pattern. The key point is that the binary choice probability is an increasing function of the ratio (δ/σ), and the ratios reproduce the correct order:

$$\begin{aligned}(\mu_B - \mu_C)/\sigma_{BC} &= +\delta/\sigma_L > (\mu_D - \mu_C)/\sigma_{DC} \\ &= +\delta/\sigma_H > (\mu_D - \mu_B)/\sigma_{DB} \\ &= 0/\sigma_L > (\mu_D - \mu_A)/\sigma_{DA} = -\delta/\sigma_H \\ &> (\mu_B - \mu_A)/\sigma_{BA} = -\delta/\sigma_L.\end{aligned}$$

In sum, including action D in a pairwise choice produces a higher variance, making it hard to discriminate the differences; whereas including option B produces a lower variance, making it easy to discriminate the differences. In this way, decision field theory provides a simple explanation for a result that is difficult to explain by a utility model.

5.2. Context effects on multi-alternative choice

Whereas Busemeyer and Townsend (1993) apply decision field theory to binary choice situations such as in the previous section, Roe et al. (2001) show how the same theory can account for decisions involving multiple alternatives. Choice behaviour becomes even more perplexing when there are more than two options in the choice set. A series of experiments have shown the preference relation between two of the options, say A and B, can be manipulated by the context provided by adding a third option to the choice set (see Rieskamp, Busemeyer, and Mellers (2006), Roe et al. (2001), Wedell (1991)).

The reference point effect (Tversky and Kahneman (1992)); see also Wedell (1991) provides a compelling example. The basic ideas are illustrated in Fig. 3, where each letter shown in the figure represents a choice option described by two conflicting objectives. In this case, option A is very good on the first objective but poor on the second, whereas option B is poor on the first objective and high on the second. When given a binary choice between options A versus B, people tend to be equally likely to choose either option. (Ignore option C for the time being).

Option R_a has a small advantage over A on the second dimension, but a more noticeable disadvantage on the first dimension. Thus R_a is relatively unattractive compared to option A. Similarly, option R_b has a small advantage over B on the first dimension but a more noticeable disadvantage on the second dimension. Thus option R_b is relatively unattractive compared to option B.

For the critical condition, individuals are presented with three options: A, B, and a third option, R, which is used to manipulate a reference point. Under one condition, participants are asked to assume that the current option R_a is the status quo (reference point), and they are then given a choice of keeping R_a or exchanging this position for either action A or B. Under these conditions, R_a was rarely chosen, and A was favoured over B. Under a second condition, participants are asked to imagine that option R_b is the status quo, and they are then given a choice of keeping R_b or exchanging this position for either A or B. Under this condition, R_b was rarely chosen again, but now B was favoured over A. Thus the preference relation between A and B reverses depending on whether the choice is made with respect to the context provided by the reference point R_a or R_b.

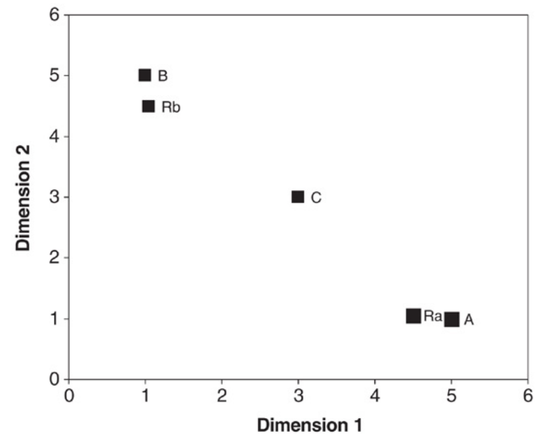

Fig. 3. Context effects can impact decisions. Option A is high on Dimension 1 but low on Dimension 2 whereas B is high on Dimension 2 but low on Dimension 1. Option C can be thought of as a compromise, and options R_a and R_b can be used as reference points. Although preference for options A and B are equivalent in binary choice comparisons, preference for A increases when R_a is present; likewise, preference for B increases when the choice set includes A, B, and R_b. This violation of regularity cannot be explained by heuristic models of choice. When presented with binary comparisons involving two of A, B, and C, preference for each option is equal. However, when all three options are simultaneously available, option C emerges as a preferred alternative, an effect which neither heuristic choice nor classic utility models can easily explain. Decision field theory can explain each of these effects without even adjusting its parameters across the conditions (see Roe et al. (2001) for other related findings).

In sum, the following pattern of choice probabilities is generally found (Tversky and Kahneman (1992); see also Wedell (1991)):

$$\begin{aligned}\Pr[A \mid \{A, B, R_a\}] &> \Pr[A \mid \{A, B\}] \\ &= .50 > \Pr[B \mid \{A, B, R_a\}],\end{aligned}$$

$$\begin{aligned}\Pr[B \mid \{A, B, R_B\}] &> \Pr[B \mid \{A, B\}] \\ &= .50 > \Pr[A \mid \{A, B, R_b\}].\end{aligned}$$

According to a traditional utility model, it seems as if option A is equal in utility to option B under binary choice, but option A has greater utility than B from the R_a point of view, and option B has greater utility than A from the R_b point of view.

There is a second and equally important qualitative finding that occurs with this choice paradigm, which is called the attraction effect (Heath & Chatterjee, 1995; Huber & Puto, 1983; Huber, Payne, & Puto, 1982; Simonson, 1989). Note that the choice probability from a set of three options *exceeds* that for the subset: $\Pr[A \mid \{A, B, R_a\}] > \Pr[A \mid \{A, B\}]$. It seems that adding the deficient option R_a makes option A look better than it appears within the binary choice context. This is a violation of an axiom of choice called the regularity principle. Violations of regularity cannot be explained by heuristic choice models such as Tversky's (1972) elimination by aspects model (see Rieskamp et al. (2006), Roe et al. (2001)).

Decision field theory explains these effects through the lateral inhibition mechanism in the feedback matrix $\boldsymbol{S}$. Consider

$$(\mathbf{I}-\boldsymbol{S}^2)^{-1}=\frac{1}{\det[\mathbf{I}-\boldsymbol{S}^2]}\cdot\begin{bmatrix}1+s^4-b^2(3-2b^2)-s^2(b^2+2) & b^2(1+3s^2-2b^2) & -2sb\cdot(1-s^2)\\ b^2(1+3s^2-2b^2) & 1+s^2-b^2(3-2b^2)-s^2(b^2+2) & -2sb\cdot(1-s^2)\\ -2sb\cdot(1-s^2) & -2sb\cdot(1-s^2) & (1-s^2)(1-s^2-2b^2)\end{bmatrix}$$

Box I.

first a choice among options $\{A, B, R_a\}$. In this case, B is very dissimilar to both A and R_a and so the lateral inhibition connection between these two is very low (say zero for simplicity); yet A is very similar to R_a and so the lateral inhibition connection between these two is higher ($b > 0$). Allowing rows 1, 2, and 3 to correspond to options A, B, R_a respectively, then the feedback matrix can be represented as

$$\boldsymbol{S}=\begin{bmatrix}s & 0 & -b\\ 0 & s & 0\\ -b & 0 & s\end{bmatrix}.$$

The eigenvalues of $\boldsymbol{S}$ in this case are $(s, s-b, s+b)$, which are required to be less than unity to maintain stability. In addition, to account for the binary choices, we set $\mu_A = \mu_B = \mu$, and $\mu_R = -2\mu$, so that A and B have equal weighted mean values, and option R_a is clearly inferior. Note that A and B are equally likely to be chosen in a binary choice, but this no longer follows in the triadic choice context. For in the latter case, we find that the mean preference state vector converges asymptotically to:

$$\xi(\infty)=h\cdot(\mathbf{I}-\boldsymbol{S})^{-1}\boldsymbol{\mu}$$
$$=h\cdot\begin{bmatrix}\dfrac{(1-s+2b)\mu}{(1-s-b)(1-s+b)}\\ \dfrac{\mu}{1-s}\\ \dfrac{(-2+2s-b)\mu}{(1-s-b)(1-s+b)}\end{bmatrix}. \tag{11}$$

The asymptotic difference between the mean preference states for A and B are obtained by subtracting the first two rows, which yields:

$$\xi_A-\xi_B=h\cdot\frac{\mu\cdot b(2(1-s)+b)}{(1-s)(1-s-b)(1-s+b)}. \tag{12}$$

This difference must be positive, producing a choice probability that favours A over B. As μ increases, the probability of choosing option R_a goes to zero, while the difference between options A and B increases, which drives the probability of choosing option A toward 1.0.

If the reference point is changed from option R_a to option R_b, then the roles of A and B reverse. The same reasoning now applies, and the sign of the difference shown in Eq. (12) reverses. Thus, if the reference point is changed to option R_b, then option B is chosen more frequently than A, producing a preference reversal. In sum, decision field theory predicts that both reference point and attraction effects result from changes in the mean preference state generated by the lateral inhibitory connections (see Busemeyer and Johnson (2004), Roe et al. (2001) for more details.)

Another important example of context effects is the compromise effect, which involves option C in Fig. 3. When given a binary choice between options A versus B, people are equally likely to choose either option. The same holds for binary choices between options A versus C, and between B versus C. However, when given a choice among all three options, then option C becomes the most popular choice (Simonson, 1989; Tversky & Simonson, 1993). Within the triadic choice context, option C appears to be a good compromise between the two extremes.

The compromise effect poses problems for both traditional utility models as well as simple heuristic choice models. According to a utility model, the binary choice results imply equal utility for each of the three options; but the triadic choice results imply a greater utility for the compromise. According to a heuristic rule, such as the elimination by aspects rules or a lexicographic rule, the intermediate option should never be chosen, and only one of the extreme options should be chosen.

Decision field theory provides a rigorous account of this effect as well. To account for the binary choices, it must be assumed that all the mean valences are equal to zero, $E[V(t)] = \boldsymbol{\delta} = \mathbf{0}$ and thus $E[P(t)] = \boldsymbol{\xi}(t) = \mathbf{0}$. This implies that the compromise effect must be explained by the covariance matrix for the triadic choice, $\boldsymbol{\Omega}(t)$. As seen in Eq. (5), this covariance matrix is generated by the lateral inhibitory connections represented by the feedback matrix $\boldsymbol{S}$. Allowing rows 1, 2, and 3 to correspond to options A, B, and C from Fig. 3, respectively, then the feedback matrix for the three options for the compromise situation is represented by:

$$\boldsymbol{S}=\begin{bmatrix}s & 0 & -b\\ 0 & s & -b\\ -b & -b & s\end{bmatrix}.$$

The eigenvalues of this matrix are $[s+\sqrt{2}\cdot b, s, s-\sqrt{2}\cdot b]$. Suppose, for simplicity, that $\boldsymbol{\Phi} = h\cdot\phi^2\cdot\mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Then $\boldsymbol{\Omega}(\infty) = h\cdot\phi^2\cdot(\mathbf{I}-\boldsymbol{S}^2)^{-1}$, where see equation in Box I. The important point to note is that the covariance between the preference states for A and B is positive, whereas the covariance between preference states for A and C is negative, and the covariance between preference states for B and C is also negative. (For example, if $s = .95$ and $b = .03$, then the correlation between A and B states is $+.56$ and the correlation between A and C states is $-.74$.) The valences vary stochastically around zero, but whenever the preference state for C happens to be strong, then the preference states for A and B are weak; whereas whenever the preference state for C happens to be weak, then the preference states for both A and B are strong. Thus when C happens to be strong, it has no competitor; but options A and B must share preference on those occasions when C happens to be weak. The result is that about half the time C will be chosen, and the remaining times either A or B will be chosen. Thus this

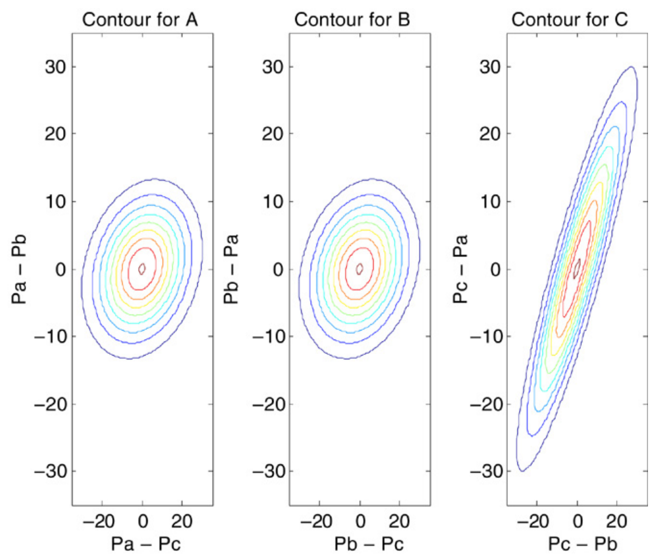

Fig. 4. Equal density contours for computing triadic choice probabilities. The leftmost panel demonstrates the contour for the density function of option A, where positive values on the horizontal axis indicate preference for option A over option C and positive values on the vertical axis indicate preference for option A over B. There is a +.22 correlation in the left panel for a choice of A, and the same is true for the middle panel (preference for B), but there is a +.93 correlation shown in the right panel for the choice of C. This figure demonstrates that when option C is preferred to option A, it is also very likely to be preferred to option B, allowing it to obtain an inordinately large share of preference, whereas preference for A over C does not indicate that it will be preferred over option B.

covariance structure provides option C with an advantage over A and B.

Fig. 4 illustrates the effect of the covariance structure on the asymptotic distribution of the differences in preference states (with $s = .95$ and $b = .03$). The panel on the left shows the equal density contours for $[P_A(\infty) - P_C(\infty), P_A(\infty) - P_B(\infty)]$: the probability that the preference state for A exceeds that for both B and C is the integral over the positive right hand quadrant in this figure, which equals .29. The middle panel illustrates the equal density contours for $[P_B(\infty) - P_C(\infty), P_B(\infty) - P_A(\infty)]$: the probability that the preference state for B exceeds that for A and C is again .29. Finally, the right panel shows the equal density contours for $[P_C(\infty) - P_B(\infty), P_C(\infty) - P_A(\infty)]$: the probability that the preference state for C exceeds that for A and B equals .43. In sum, decision field theory predicts the compromise effect as a result of the covariance structure generated by the lateral inhibitory connections (see Roe et al. (2001), for more details).

5.3. *Deliberation time effects on choice*

Decisions take time, and the amount of time allocated to making a decision can change preferences. Decision field theory has been successful in accounting for a number of findings concerning the effects of deliberation time on choice probability (see Busemeyer (1985), Diederich (2003), Dror, Busemeyer, and Basola (1999)). Busemeyer and Townsend (1993) discuss the ability of decision field theory to account for speed–accuracy tradeoffs that are evident in many choice

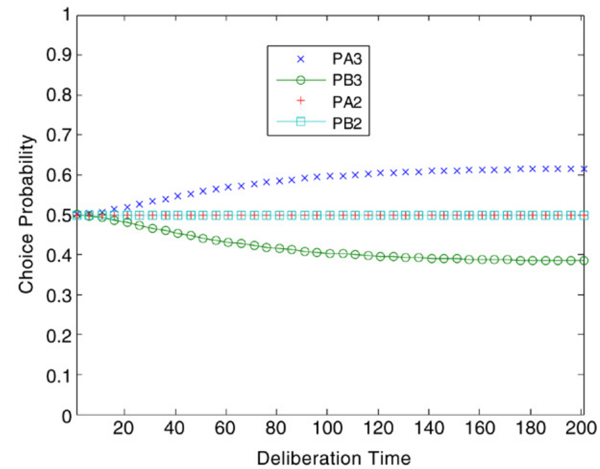

Fig. 5. Predictions for the attraction effect as a function of deliberation time. $PA2$ and $PB2$ indicate choice probability as a function of deliberation time for options A and B, respectively (from Fig. 3), in the binary choice condition whereas $PA3$ and $PB3$ demonstrate preference for Options A and B, respectively, during triadic choice when option R_a is included in the choice set. Decision field theory predicts that the attraction effect should increase with deliberation time, an effect empirically demonstrated Dhar et al., 2000; Simonson, 1989.

situations. That is, in general, choice probabilities are moderated with decreases in deliberation time.

As an example, consider once again the attraction effect referred to in Fig. 3: Adding a third option R_a to the choice set increases the probability of choosing option A. Decision field theory predicts that increasing the deliberation time increases the size of the attraction effect. In fact, this prediction has been confirmed in several experiments (Dhar, Nowlis, & Sherman, 2000; Simonson, 1989). In other words, thinking longer actually makes people produce stronger violations of the regularity axiom.

Eq. (12) represents the asymptote of an effect that is predicted to grow during deliberation time. The dynamic predictions of the model were computed assuming an externally controlled stopping task, with increasing values for the stopping time T. The predictions were generated using the coordinates of options A, B, and R_A shown in Fig. 3 to define the values, m_{jk}, for each option on each dimension, $w_k = .50$ for the mean attention weight allocated to each dimension, $s_{ii} = .95$ for the self feedback, $s_{AR} = -.03$ for the lateral inhibition between the similar options A and $R_a, s_{AB} = 0$ for the lateral inhibition between the two dissimilar options A and B, and $h = 1$. The predicted choice probabilities are plotted in Fig. 5. The choice probabilities for the binary choice lie on the .50 line; the probability of choosing option A from the triadic choice gradually increases from .50 to above .60; the probability of choosing option B from the triadic choice gradually decreases from .50 to below .40; and the probability of choosing option R_a remains at zero. As can be seen in this figure, the model correctly predicts that the attraction increases with longer deliberation times.

Decision field theory accounts for not only moderation of preference strength, but even reversals in preference among options as a function of deliberation time. Specifically, Diederich

(2003) found reversals in pairwise choices under time pressure, and demonstrates the ability of decision field theory to account for her results. Decision field theory requires an initial bias ($z \neq 0$) to produce such reversals, such that the initial preference favours one alternative whereas the valence differences tend to favour the other alternative. In this case, it requires time for the accumulated valences to overcome the initial bias. Alternatively, specific forms of attention switching not discussed here can be used instead to predict the results (see Diederich (1997, 2003), for model details). Utility theories, as static accounts of decision making, are not able to make reasoned predictions regarding the effects of time pressure whatsoever.

5.4. *Choosing not to choose*

Recently, Busemeyer, Johnson, and Jessup (2006) addressed a new phenomenon concerning context effects reported by Dhar and Simonson (2003) that occur when an option to 'defer making a decision' is included in the choice sets. Dhar and Simonson (2003) found that adding a deferred option had opposite effects on the attraction and compromise effects—it increased the attraction effect, and it decreased the compromise effect.

Busemeyer et al. (2006) showed that decision field theory is able to account for these new effects using the same model specifications made for the original compromise and attraction effects reported earlier. The only additional assumption is that the option to defer is treated as a new choice option with values equal to the average of the values for the presented options. This assumption produced a predicted increase in the attraction effect (by 10%) that resulted from the deferred option stealing probability away from the original options in the binary choice set. At the same time, this assumption produced a predicted decrease in the compromise effect (by 7%) that resulted from the deferred option decreasing the advantage of compromise option over the other two extreme options in the triadic set.

5.5. *Preference eeversals between choice and price measures*

One important advantage of decision field theory is that it is not limited to choice-based measures of preference, and it can also be extended to more complex measures such as prices. This is important because empirically it has been found that preference orders can reverse across choice and price measurements (Lichtenstein and Slovic (1971), see Johnson and Busemeyer (2005) for a review). The basic finding is that low-variance gambles are chosen over high-variance gambles of similar expected value, whereas the high-variance gamble receives a higher price. Furthermore, researchers have also found discrepancies between buying and selling prices that can be large enough to produce preference reversals. Here as well, the buying price is greater for the low-variance gamble compared to the high-variance gamble, but the selling prices produce the opposite rank-order (see Johnson and Busemeyer (2005), for a review).

Johnson and Busemeyer (2005) propose that reporting a price for a gamble results from a series of implicit comparisons between the target gamble and a set of candidate prices. Each comparison is modelled by an implicit binary comparison process, where an implicit choice favouring the current candidate price entails decreasing the candidate price for the next comparison, and an implicit choice favouring the target gamble entails increasing the candidate price for the next comparison. The comparison process continues until a price is found that is considered equal to the target gamble—that is, a price which produces indifference ($P(t) = 0$) when compared with the target gamble. Johnson and Busemeyer (2005) show that this model accounts for a collection of response mode effects that no other theory has been shown to successfully predict, including both types of preference reversals mentioned above.

6. Alternate neural network models for complex decisions

Several artificial neural network or connectionist models have been recently developed for judgment and decision tasks (Grossberg & Gutowski, 1987; Guo & Holyoak, 2002; Holyoak & Simon, 1999; Levin & Levine, 1996; Read, Vanman, & Miller, 1997; Usher & McClelland, 2004). The Grossberg and Gutowski (1987) model was used to explain preference reversals between choice and prices (Section 5.5), but it has never been applied to the phenomena discussed in Sections 5.1–5.4. The Levin and Levine (1996) model can account for effects of time pressure on choice (Section 5.3), but it has not been applied directly to any of the other phenomena reviewed here. The model by Holyoak and colleagues has been applied to attraction effects, but it cannot account for similarity effects for binary choices (Section 5.1), nor can it account for compromise effects for triadic choices, and it has not been applied to preference reversals between choice and prices (Section 5.5). The Read et al. (1997) model was designed for reasoning rather than preference problems, and so it is not directly applicable to the phenomena reviewed here. Finally, the Usher and McClelland (2004) model can account for similarity effects (Section 5.1), context effects (Section 5.2), and time pressure effects (Section 5.3), but it has not been applied to the remaining phenomena. A closer comparison with the Usher and McClelland (2004) model is presented below.

Usher and McClelland (2004) have recently proposed the leaky competing accumulator (LCA) model that shares many assumptions with decision field theory, but departs from this theory on a few crucial points. This model makes different assumptions about (a) the dynamics of response activations (what we call preference states), and (b) the evaluations of advantages and disadvantages (what we call valences). First, they use a nonlinear dynamic system that restricts the response activation to remain positive at all times, whereas we use a linear dynamical system that permits positive and negative preference states. The non-negativity restriction was imposed to be consistent with their interpretation of response activations as neural firing rates. Second, they adopt Tversky and Kahneman's (1991) loss aversion hypothesis so that disadvantages have a larger impact than advantages. Without the assumption of loss aversion, their theory is unable to account for the attraction and compromise effects discussed in Section 5.2. Furthermore, no explanation is given regarding the behavioural emergence of loss aversion from underlying neurophysiology. Third, the lateral inhibition employed by LCA is not distance dependent.

It is interesting to consider why LCA does not incorporate distance dependent lateral inhibition. An essential feature of neural networks is their use of distributed activation patterns for representations, with similar representations sharing more activation pattern overlap than dissimilar representations (Rumelhart & McClelland, 1986). Thus, neural networks maintain distance dependent activation; consequently, it is only natural that they would possess distance dependent inhibition as well. And, as has already been stated, the striatum, a key substructure in the decision making process, is itself a network of distance dependent laterally inhibitory neurons. So, distance dependent inhibition is more neurally plausible than not. As to why LCA does not incorporate distant dependent inhibition, it might be due to the notion that, coupled with the inverse S-shaped value function used to produce loss aversion, the model would be severely hampered in its ability to account for the compromise effect.

Usher and McClelland (2004) criticized Roe et al. (2001) because the latter model allows for both negative as well as positive preference states, which they argue is inconsistent with the concept of strictly positive neural activation. Busemeyer et al. (2005) responded that the zero point of the preference scale can be interpreted as the baseline rate of activation in neural model, with negative states representing inhibition below baseline, and positive activation representing excitation above baseline. However, Busemeyer et al. (2005) also formulated a nonlinear version of lateral inhibition with strictly positive states which is also able to account for the context effects discussed in Section 5.2 (see Fig. 6). Specifically, they assumed that:

$$dP_j(t+h) = s_{ii} \cdot P_j(t) + V_j - \sum_{i \neq j} s_{ij} \cdot [P_i(t) - b], \tag{13}$$

$$P_j(t+h) = F[P_j(t) + dP_j(t+h)], \quad \text{and}$$

$$F(x) = 0 \quad \text{if } x < 0, \qquad F(x) = x \quad \text{if } x \geq 0.^{4} \tag{14}$$

As shown in Fig. 6, this version of a lateral inhibitory network still reproduces the attraction effect while only using positive activation and positive input states.

A final criticism of decision field theory levied by Usher and McClelland (2004) is that the model utilizes linear dynamics even though the brain is most certainly a nonlinear system. As Busemeyer et al. (2005) argued, one reason that decision field theory has retained linear dynamics is that mathematical solutions can be obtained for the model. And because linear dynamics can approximate nonlinear systems, this increased mathematical tractability comes at a minimal cost. No mathematical solutions have been derived for the nonlinear Usher and McClelland (2004) model, and therefore one must rely on computationally intensive Monte Carlo simulations.

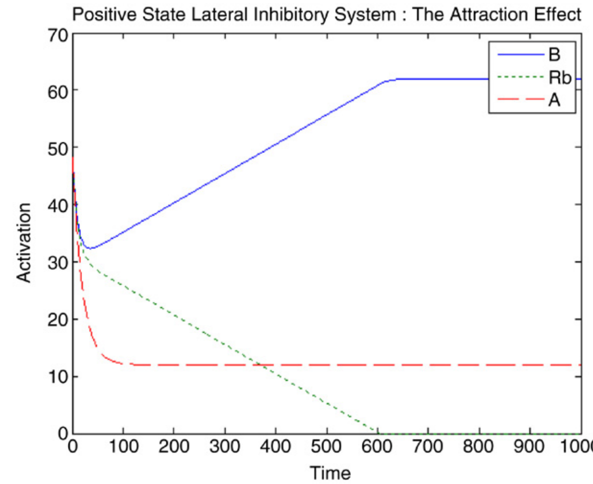

Fig. 6. Choice probabilities as a function of deliberation time (units on time scale are arbitrary) for options A, B, and R_b from Fig. 3, derived from a nonlinear implementation of decision field theory that is constrained to maintain positive activation states. The nonlinear version still reproduces the attraction effect without appealing to loss aversion.

This issue of tractability becomes extremely important when trying to scale up models to account for more complex measures of preference such as prices. Computationally, it would be very difficult to model such complex process using brute force simulations.

7. Conclusion

Models of human behaviour exist on a variety of explanatory levels, tailored to different aspects of behaviour. In the field of decision making, the most popular models for many decades have been algebraic utility models developed in economics. These models may serve as a good first approximation to macro-level human behaviour, but are severely limited in that they only attempt to describe decision outcomes. Furthermore, their static and deterministic nature does not allow them to account for decision dynamics and response variability, respectively.

Recently, biologically-inspired models have been developed that focus on the substrates of overt decision behaviour. These models capture the dynamics of neural activation that give rise to simple decisions underlying sensorimotor responses. Despite growing interest in neuroscience among economists and decision researchers, these micro-level models have not yet had a profound impact. Unfortunately, the majority of neuroscience by decision researchers only studies gross brain activation during traditional tasks that is then somehow related to existing aggregate-level models.

Here, we have provided a level of analysis that we believe shows excellent potential in bridging the gap between the customary approach of decision researchers and the contemporary advances in neuroscience. We introduced the fundamental concepts of modelling decision making via diffusion processes, based on decision field theory. This approach models directly the deliberation process that results in overt choice, in line with neural models and in contrast to

4 Where the usage of F corresponds to that utilized by LCA.

algebraic utility theories. Decision field theory has also been applied to 'higher-order' cognitive tasks such as multi-attribute, multi-alternative choice and pricing, as have utility theories (but not existing neural models of sensorimotor decisions). Thus, diffusion models such as decision field theory seem to offer the best of both worlds from a modelling standpoint.

Emergent properties of decision field theory allow it to explain systematic changes in preference that have challenged the prevailing utility framework. Examples reviewed here included applications to changes in the choice set or response method, and dependencies on deliberation time. Decision field theory has also been shown to account for a number of other pervasive idiosyncrasies in human decision behaviour (see Busemeyer and Johnson (2004)).

Other models in the same class as decision field theory, but relying on other specific processes, were reviewed as well. Each of these alternatives has specific advantages and disadvantages, and they do not always make the same predictions in a given situation. Further research is needed to discriminate between these various 'bridge' models of decision making. Regardless of exactly which model is determined to be the most successful tomorrow, this type of modelling in general delivers superior theoretical benefits today.

Acknowledgements

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